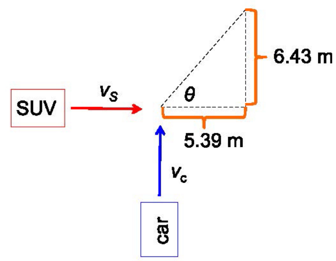
Appendix

The following question is a typical example of the kinds of quantitative questions students in the reformed section were asked to solve during the lecture portion of class. Students were given the following prompt and diagram (which is based on exercise 8.41 in Young & Freedman [2012]) and asked to solve the problem while working collaboratively with their neighbors:



Prompt and diagram

> A car of mass m_c =1500 kg is traveling north through an intersection when it is hit by an SUV of mass m_s = 2200 kg traveling east. The two vehicles become locked together during the impact and slide together as one after the collision. The cars slide to a halt at a point 5.39 m east and 6.43 m north of the impact point. The coefficient of kinetic friction between the tires and the road is μ_k = 0.75. What were the speed of the car (v_c) and the speed of the SUV (v_s) just before the impact?

The first-time instructor would circulate around the room for several minutes while students worked, answering questions from individual groups. He would then bring the entire class back together and ask the following TPS questions in order to focus attention on and help students overcome particularly challenging aspects of the solution pathway.

TPS question 1

If W_f is the work done by kinetic friction while the two connected cars slide across the ground, then which of the following is true?
(A) $W_f = (\frac{1}{2} m_s v_s^2 + \frac{1}{2} m_c v_c^2) - \frac{1}{2}(m_s + m_c)(v_{s+c})^2$
(B) $W_f = (\frac{1}{2} m_s v_s^2 + \frac{1}{2} m_c v_c^2)$
(C) $W_f = 0 - \frac{1}{2}(m_s + m_c)(v_{s+c})^2$
(D) $W_f = (\frac{1}{2} m_s v_s^2 + \frac{1}{2} m_c v_c^2) + \frac{1}{2}(m_s + m_c)(v_{S+c})^2$

Commentary: Students must realize that they must apply both the work-energy theorem and the conservation of linear momentum in order to solve for the two unknown speeds. This question makes students think about how the work-energy theorem applies in the case of a completely inelastic collision.

TPS question 2

What is the work done by kinetic friction?

(A) $W_f = -\mu_k (m_s + m_c)\, g\, [(6.43\ \mathrm{m})^2 + (5.39\ \mathrm{m})^2]^{1/2}$
(B) $W_f = -\mu_k (m_s + m_c)\, g\, [(6.43\ \mathrm{m})^2 + (5.39\ \mathrm{m})^2]$
(C) $W_f = -\mu_k (m_s + m_c)\, g\, (6.43\ \mathrm{m} + 5.39\ \mathrm{m})$
(D) $W_f = -\mu_k (m_s + m_c)\, g\, (6.43\ \mathrm{m})$
(E) $W_f = -\mu_k (m_s + m_c)\, g\, (5.39\ \mathrm{m})$

Commentary: This question helps students reason about the distance the two vehicles travel after the collision, which is necessary for calculating the work done by the nonconservative friction force. The answer choices for this question deliberately blend symbolic notation and specific numbers in order to facilitate students' abilities to efficiently reason through the five options.

TPS question 3

Which of the following is the conservation of linear momentum applied to the *x*-components of the momenta:

(A) $m_s v_s + m_c v_c = (m_c + m_s)\, v_{s+c}$

(B) $m_s v_s + m_c v_c = (m_c + m_s)\, v_{s+c} \tan(\theta)$

(C) $m_s v_s = (m_c + m_s)\, v_{s+c} \cos(\theta)$

(D) $m_c v_c = (m_c + m_s)\, v_{s+c} \sin(\theta)$

(E) more than one of the above

Commentary: This question requires students to think about the conservation of linear momentum and how it can be used to relate the speed of the vehicles after they collide to their precollision speeds. This question also forces students to reason about which trigonometric function they need to use, which is a frequent source of confusion among introductory physics students.

TPS question 4

Which of the following is the conservation of linear momentum applied to the *y*-components of the momenta:

(A) $m_s v_s + m_c v_c = (m_c + m_s)\, v_{s+c}$

(B) $m_s v_s + m_c v_c = (m_c + m_s)\, v_{s+c} \tan(\theta)$

(C) $m_s v_s = (m_c + m_s)\, v_{s+c} \cos(\theta)$

(D) $m_c v_c = (m_c + m_s)\, v_{s+c} \sin(\theta)$

(E) more than one of the above

Commentary: This question is the same as TPS question 3, except for the *y*-components.

TPS question 5

What is cos(θ)?
(A) 5.39 / 6.43
(B) 6.43 / 5.39
(C) 5.39 / (5.39 + 6.43)
(D) 6.43 / (5.39 + 6.43)
(E) 5.39 / $(5.39^2 + 6.43^2)^{1/2}$

Commentary: Students sometimes need to be reminded of the definition of trigonometric quantities in terms of the sides of a right triangle.

After this final question, students were given a few more minutes to calculate a solution. The first-time instructor then asked for the class to shout out the initial speed of the car and the SUV before revealing the solution:

Solution

First apply the work-energy theorem:

$$v_{S+C}^2 = 2\mu_k g\sqrt{(6.43\ \text{m})^2 + (5.39\ \text{m})^2} = 123\ \text{m}^2/\text{s}^2$$
$$v_{S+C} = 11.1\ \text{m/s}$$

Next apply the conservation of momentum:

$$v_c = \frac{(m_c + m_s)v_{s+c}\sin(\theta)}{m_c} = \frac{(1500\ \text{kg} + 2200\ \text{kg})(11.1\ \text{m/s})}{1500\ \text{kg}}\frac{6.43}{\sqrt{6.43^2 + 5.39^2}} = 21.0\ \text{m/s}$$
$$v_s = \frac{(m_c + m_s)v_{s+c}\cos(\theta)}{m_s} = \frac{(1500\ \text{kg} + 2200\ \text{kg})(11.1\ \text{m/s})}{1500\ \text{kg}}\frac{5.39}{\sqrt{6.43^2 + 5.39^2}} = 12.0\ \text{m/s}$$